Schrödinger’s Register:
Foundational Issues and Physical Realization

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Abstract—This work-in-progress paper consists of four points which relate to the foundations and physical realization of quantum computing. The first point is that the qubit cannot be taken as the basic unit for quantum computing, because not every superposition of bit-strings of length n can be factored into a string of n-qubits. The second point is that the “No-cloning” theorem does not apply to the copying of one quantum register into another register, because the mathematical representation of this copying is the identity operator, which is manifestly linear. The third point is that quantum parallelism is destroyed if a processing step does not violate the “No-cloning” theorem. The fourth point is that the time to environmental decoherence is inversely related to the size of the physical system considered. If this is so, then the longer the q-string, the shorter the time to its environmental decoherence. This rules out quantum parallelism for q-strings of arbitrary length. Besides environmental decoherence, two other kinds of decoherence are introduced and discussed.

II. ELABORATIONS OF THE FOUR POINTS

A. Qubits and Q-Strings

A bit is 0 or 1. A string of bits of length n = |b1 . . . bn⟩. The number of all strings of bits of length n = 2n. A q-string of length n is a sum of the bit strings of length n weighted by complex numbers. So, a q-string of length n = |ψ⟩ = ∑(m=1 to n) c_m |φ_m⟩,

where the φ_m are the bit strings of length n and the complex numbers c_m satisfy the condition ∑(m=1 to n) |c_m|^2 = 1. Some of the c_m may be 0, so it may be that not every bit string of length n is a nonzero-weighted component of the q-string. A qubit q_i = a q-string of length 1. So, q_i = α_i |0⟩ + β_i |1⟩, where |α_i|^2 + |β_i|^2 = 1. A string of qubits is a product of qubits = q_1 ⊗ q_2 ⊗ . . . q_n.

Consider a string of two qubits:

q_1 ⊗ q_2 = (α_1 |0⟩ + β_1 |1⟩) ⊗ (α_2 |0⟩ + β_2 |1⟩) = α_1α_2 |00⟩ + α_1β_2 |01⟩ + β_1α_2 |10⟩ + β_1β_2 |11⟩.

|α_1|^2 + |β_1|^2 = 1 and |α_2|^2 + |β_2|^2 = 1.

Now consider the q-string of length 2:

0 |00⟩ + 1/√2 |01⟩ + 1/√2 |10⟩ + 0 |11⟩
If this q-string = \( q_1 \otimes q_2 \), then \( \alpha_1\alpha_2 = 0 \), \( \alpha_1\beta_2 = \frac{1}{\sqrt{2}} \), \( \beta_1\alpha_2 = \frac{i}{\sqrt{2}} \), and \( \beta_1\beta_2 = 0 \). So, either \( \alpha_1 = 0 \) or \( \alpha_2 = 0 \). But if \( \alpha_1 = 0 \), then \( \alpha_1\beta_2 = 0 \neq \frac{1}{\sqrt{2}} \). If \( \alpha_2 = 0 \), then \( \beta_1\alpha_2 = 0 \neq \frac{1}{\sqrt{2}} \). So the above q-string of length 2 is not a string of 2 qubits. This argument can be generalized. So, not every q-string of length \( n \) is a string of \( n \) qubits. The above q-string of length 2 is said to have “entangled qubits”, because it cannot be factored into a string of 2 qubits.

**C. Quantum Parallelism and Decoherence**

What is quantum parallelism? Suppose \( \psi \) is a q-string and \( \psi \) has \( m \) different bit strings appearing as nonzero-weighted components. Then, quantum parallelism is the idea that one processing step on \( \psi \) is, in a sense, equivalent to \( m \) processing steps on the bit string components of \( \psi \) [2]. Since processing takes time, quantum parallelism is lost if the q-string decoheres during processing. What is decoherence? The only kind of decoherence discussed as such in quantum computing is environmental decoherence. We believe that there are two other forms of decoherence, measurement decoherence and internal decoherence, and that these other forms may pose obstacles for quantum parallelism as well. Let us start with environmental decoherence. Let \( \psi = \sum_m c_m \phi_m \) be the state of a physical system where the \( \phi_m \) are the base states of the system and the \( c_m \) are the complex numbers satisfying the usual condition. Let \( E_0 \) represent the initial state of the environment. Environmental decoherence is the idea that after a time (decoherence time) the physical system interacts enough with the environment so that the state of the system plus environment evolves to the following:

\[
| \psi, E_0 \rangle \longrightarrow \sum_m c_m | \phi_m, E_m \rangle,
\]

where the \( E_m \) are states of the environment that do not mutually “interfere”. What the “non-interference” means practically is that the evolved state immediately collapses:

\[
\sum_m c_m | \phi_m, E_m \rangle \longrightarrow \text{one of the } | \phi_m, E_m \rangle \text{ states}
\]

with a probability of \( | c_m |^2 \). Q-strings live on the register of the quantum computer. So, \( \Psi \) above is the state of the register(s) of the computer, and \( E \) above is the state of the environment of the register(s); i.e., the rest of the computer plus the external world. So, if decoherence time is less than processing time, a q-string will collapse into one of its component bit strings, and quantum parallelism will be destroyed. Erich Joos [1] states that experimental results seem to indicate that decoherence time is related inversely to size; he even says (p. 13): “...macroscopic objects are extremely sensitive and immediately decohered.” If what Joos says is true, then the longer the q-string, the shorter the time to its decoherence. This rules out quantum parallelism for q-strings of arbitrary length. Joos says (p. 14):

...(decoherence) represents a major obstacle for people trying to construct a quantum computer. Building a really big one may well turn out to be as difficult as detecting other Everett worlds!

Many think that detecting other Everett worlds is impossible [3]. Measurement decoherence can be explained as follows.
Let
\[ \psi = \sum m c_m \phi_m \]
be the state of a physical system, and suppose at \( t_0 \) (the initial time), \( \psi \) is coupled with a measuring device in state \( M_0 \). Let “measurement time” be the amount of time required for the measuring device to measure the physical system, i.e., the amount of time for the measuring device to evolve from \( M_0 \) to a superposition of indicator states \( M_i \). The picture of the evolution is as follows:

\[ \sum_m c_m \phi_m M_0 \rightarrow \sum_m c_m \phi_m M_m. \]

If we make the assumption that the \( M_i (i \geq 1) \) do not mutually interfere, then \( \psi \) immediately collapses:

\[ \sum_m c_m \phi_m M_m \rightarrow \text{one of the } \phi_m \text{ with a probability of } |c_m|^2. \]

Measurement decoherence (called quantum measurement by quantum computer scientists) is a resource of, and not an obstacle to, quantum computing if it occurs after processing is complete. Measuring the output q-string is the way to read information contained in that q-string. No one will intentionally apply a measuring device to a register or registers before processing is complete. So, how can measurement decoherence be an obstacle to quantum computing? A physical quantum computer will contain a register or registers, but will also contain other devices (for processing, etc.) besides registers. If the “innards” of a physical quantum computer exclusive of the registers act like a measuring device during processing, then there will be an unintentional measurement of a register or registers during processing, and quantum parallelism will be lost. Thus, it is a challenge not only to build registers that can exist in superposed states, but also to build the rest of the quantum computer so that it does not act like a measuring device on registers during processing.

The third form of decoherence is internal decoherence. Suppose we have a physical system as an initial state \( \psi_0 \). Suppose also that in some time interval (evolution time), the physical system evolves to a superposition of base states:

\[ \psi \rightarrow \sum_m c_m \phi_m. \]

If we assume that the \( \phi_m \) do not mutually interfere, we have immediate collapse:

\[ \sum_m c_m \phi_m \rightarrow \text{one of the } \phi_m \text{ with a probability of } |c_m|^2. \]

In the standard two-slit experiment, we have evolution to:

\[ \alpha \mid \text{particle travels through slit 1}\rangle + \beta \mid \text{particle travels through slit 2}\rangle, \]

but these two states mutually interfere, as is evidenced by the interference pattern built up on the photographic backstop as the experiment is repeated. So the standard two-slit experiment is not an example of internal decoherence.

We can get internal decoherence if we modify the two-slit experiment. Put a light source near slit 1, so that a particle traveling through slit 1 produces a light flash because a photon from the source bounces off the particle. Then we have evolution to:

\[ \alpha \mid \text{particle travels through slit 1 + flash of light}\rangle + \beta \mid \text{particle travels through slit 2 + no flash of light}\rangle. \]

These two states do not mutually interfere, as is evidenced by the lack of interference pattern on the photographic backstop. (Remember that observation of the light flash is not necessary to destroy interference; only existence of the flash is necessary.

Another physical system that internally decoheres is Schrödinger’s Cat Box, consisting of a box occupied by a radioactive source, Geiger counter, trip hammer, vial of cyanide, and live cat. The box evolves into:

\[ \alpha \mid \text{dead cat, smashed vial, tripped hammer, etc.}\rangle + \beta \mid \text{live cat, unsmashed vial, untripped hammer, etc.}\rangle. \]

Based on all available observational evidence, these two states do not mutually interfere. No one has ever observed a superposition of \( \alpha \mid \text{dead cat}\rangle + \beta \mid \text{live cat}\rangle \), let alone:

\[ \alpha \mid \text{smashed vial}\rangle + \beta \mid \text{unsmashed vial}\rangle, \]
\[ \alpha \mid \text{tripped hammer}\rangle + \beta \mid \text{untripped hammer}\rangle, \]

So, the solution to Schrödinger’s Paradox is internal decoherence. We can talk of Schrödinger’s Register instead of Schrödinger’s Cat, and mean by this an \( n \)-bit register that can exist in a superposition of bit strings of length \( n \) such that those bit strings do interfere. (We want the bit strings to interfere, or else we would have a collapse to a single bit string and no quantum parallelism.) So, the challenge for quantum computer scientists is to build Schrödinger’s Register. Good luck!

**D. Qubits, Q-Strings, and Entanglement**

Consider a q-string of length \( n, \psi \). Suppose \( \psi \) is “entangled.” Then it is not equal to a string of \( n \) qubits, but a string of \( n \) qubits can be constructed from it in the following way:

Survey the bit string components of \( \psi \). Let \( m \) be a position from 1 to \( n \) in the bit string. Add the amplitudes for all components with a 0 in position \( m \). Call the sum \( \alpha_m \). Add the amplitudes for all components with a 1 in position \( m \). Call the sum \( \beta_m \). Construct the qubit \( (\alpha_m |0\rangle + \beta_m |1\rangle) \).

Take the product of such qubits for all positions. This is the
string of qubits to be constructed:

\[
\bigotimes_{m=1}^{m=n} (\alpha_m | 0 \rangle + \beta_m | 1 \rangle) = \bigotimes_{m=1}^{m=n} q_m.
\]

The result of processing \(\psi \) “one qubit at a time” = the product of the results of applying a processing step \(f\) on qubits to each qubit in the string constructed from (but not identified with) \(\psi\):

\[
\bigotimes_{m=1}^{m=n} f(q_m) = \bigotimes_{m=1}^{m=n} q'_m.
\]

A string of \(m\) qubits has no entanglement among qubits. So, processing \(\psi\) “one qubit at a time” destroys entanglement.

III. CONCLUSION

We believe that the four points above will all be necessary to progress in understanding how to realize a quantum computer. Most fundamental would be the shift from qubit to q-string as the basic entity of quantum computation. At the same time it is important to see that “No Cloning” is not necessarily an obstacle to the moving of data between one register and another in a quantum computer, a necessary to realizing practical quantum computation. Also, we show that there are a number of types of decoherence, and that understanding the difference between these allows us to build a quantum register whose processes are relatively stable in the external environment of a real physical machine.

In our future research, we plan to explore more deeply the properties of a q-string versus a string of qubits and to present a more formal proof of the difference between these two concepts. We shall study the notion of entanglement and how it relates to quantum parallelism. In particular, since decoherence is the major obstacle to achieving quantum parallelism, we would like to understand better the relationship between entanglement and decoherence. In addition, we want to make clear how “No Cloning” affects the design and implementation of quantum memory and storage.

REFERENCES


