The Kind of Studies We Can’t Do Anymore

- Negative operant conditioning with a random reward system
- Addictive behavior under a random reward system
FBJ murine osteosarcoma viral oncogene homolog B
Tali’s Weird Scoring System

(1,3,4,6) : Venus -- all different sides.
(6,6,6,4) : Total = 22
(6,6,6,3) : Total = 21
(6,6,4,4) : Total = 20
(6,6,6,1) : Total = 19 (high)
(6,6,4,3) : Total = 19
(6,6,3,3) : Total = 18
(6,6,4,1) : Total = 17
(6,6,3,1) : Total = 16
(4,4,4,3) : Total = 15
(6,6,1,1) : Total = 14 (high)
(4,4,3,3) : Total = 14
(4,4,4,1) : Total = 13
(4,4,3,1) : Total = 12

(4,3,3,1) : Total = 11
(4,4,1,1) : Total = 10 (high)
(3,3,3,1) : Total = 10
(4,3,1,1) : Total = 9
(3,3,1,1) : Total = 8
(4,1,1,1) : Total = 7
(3,1,1,1) : Total = 6

(6,6,6,6) : Vultures -- all same
(4,4,4,4) : Vultures -- all same
(3,3,3,3) : Vultures -- all same
(6,x,x,x) : Senio -- a single six and anything
(6,6,6,6) : Vultures -- all same

(6,6,6,6) : Vultures -- all same
(4,4,4,4) : Vultures -- all same
(3,3,3,3) : Vultures -- all same
(1,1,1,1) : Dogs -- lowest of the Vultures
Consider

- There are 3,141 counties in the US
- A study reveals that the counties with the lowest incidence of kidney cancer are mostly rural, sparsely populated, and located in traditionally Republican-leaning states in the South, Midwest, and West
Consider

• There are 3,141 counties in the US

• A study reveals that the counties with the highest incidence of kidney cancer are mostly rural, sparsely populated, and located in traditionally Republican-leaning states in the South, Midwest, and West
The Predictability of Randomness

• Imagine a large urn filled with marbles – half red, half blue

• Two (patient) people draw marbles from the urn, record when the drawn sample is homogenous, return the marbles and repeat.

• Someone who draws marbles four at a time will get a homogenous sample at a rate of ~12.5%, while someone who draws seven at a time has a likelihood of ~1.56%
The Predictability of Randomness

• Eight babies are born one-after-the-other in a hospital

• The events are independent of each other – the number of boys and girls who were born in the hospital in the past few hours has no effect whatsoever on the sex of the next baby

• Which sequence is more likely?
  • BBBBGGGGGG
  • GGGGGGGGGGG
  • BGBBGBPGBB
One Die

- 1 in 6
Two Dice?
Birth of Probability Theory

- Antoine Gombaud, Chevalier de Méré, writes to Blaise Pascal in 1654
- Does well betting that he’ll roll at least one “ace” (1) in 4 throws. After all, $1/6 \times 4 = 4/6$
- Friends got tired of losing to him, so he comes up with a new game. Figures he’ll do just as well rolling at least one “double ace” in 24 throws. After all, $1/36 \times 24 = 24/36$
- Not quite working out as planned
The 10 Basic Rules

1) Fractions are Decimals are Percents
   • Just different ways of representing the same number
   • $\frac{1}{2} = 0.5 = 50\%$
   • Percent $\rightarrow$ Per Cent $\rightarrow$ For each 100
The 10 Basic Rules

2) Ranges from 0 to 1

- 0% to 100%
- No, you actually can’t give 110% effort
- What if the Chevalier had 7 dice? 7/6 chance of winning?
The 10 Basic Rules

3) Probability = “desired” ÷ “possible”

• Rolling a 6 on one die? $1/6 \approx .167 = 16.7\%$
• Rolling an even number? $3/6 = .5 = 50\%$
4) Enumerate

• Sometimes finding “desired” and “possible” isn’t trivial

• If you flip a coin three times, what’s the probability of getting at least two “heads”?

• \( HHH, HHT, HTH, HTT, THH, THT, TTH, TTT \)
The 10 Basic Rules

5) Actually Roll the Dice

• Theoretical odds and measured outcomes don’t always line up

• Fast and easy to simulate millions of die throws
The 10 Basic Rules

6) “Or” means add. Sometimes.

- The events must be mutually exclusive, meaning that both cannot happen simultaneously

- Odds of drawing a Jack or a 6?

  - $\frac{4}{52} + \frac{4}{52} = \frac{2}{13}$

- Odds of drawing a Jack or a spade?
The 10 Basic Rules

7) “And” means multiply. Sometimes.

- The events must not be mutually exclusive, meaning that they must be able to happen simultaneously.

- Odds of rolling two 6s? \( \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \)

- Odds of drawing a Jack and a spade? \( \frac{4}{52} \times \frac{13}{52} = \frac{1}{52} \)
The 10 Basic Rules

8) 1 - “happens” = “doesn’t happen”
   • Back to the Chevalier’s problem
   • Odds of snake eyes in 24 throws? A pain to calculate, but it’s easy to determine the odds of it not happening
     • 24 non-snake-eye rolls = \((35/36)^{24}\) = .5086
     • The Chevalier wins 49.14% of the time
     • He was winning 1 - \((5/6)^4\) = 51.77% with his first game
9) The Sum of Multiple Linear Random Selections Is Not a Linear Random Selection

- Adding uniform distributions (equally likely results) do not add up to another uniform distribution

- This was the Chevalier’s mistake
## Two Dice

<table>
<thead>
<tr>
<th>Total</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcomes</td>
<td>1-1</td>
<td>1-2</td>
<td>1-3</td>
<td>1-4</td>
<td>1-5</td>
<td>1-6</td>
<td>2-6</td>
<td>3-6</td>
<td>4-6</td>
<td>5-6</td>
<td>6-6</td>
</tr>
<tr>
<td></td>
<td>2-1</td>
<td>2-2</td>
<td>2-3</td>
<td>2-4</td>
<td>2-5</td>
<td>3-5</td>
<td>4-5</td>
<td>5-5</td>
<td>6-5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3-1</td>
<td>3-2</td>
<td>3-3</td>
<td>3-4</td>
<td>4-4</td>
<td>5-4</td>
<td>6-4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4-1</td>
<td>4-2</td>
<td>4-3</td>
<td>5-3</td>
<td>6-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5-1</td>
<td>5-2</td>
<td>6-2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Favorable</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Probability</td>
<td>1/36</td>
<td>1/18</td>
<td>1/12</td>
<td>1/9</td>
<td>5/36</td>
<td>1/6</td>
<td>5/36</td>
<td>1/9</td>
<td>1/12</td>
<td>1/18</td>
<td>1/36</td>
</tr>
<tr>
<td>Percentage</td>
<td>2.78</td>
<td>5.56</td>
<td>8.33</td>
<td>11.1</td>
<td>13.4</td>
<td>16.7</td>
<td>13.4</td>
<td>11.1</td>
<td>8.33</td>
<td>5.56</td>
<td>2.78</td>
</tr>
</tbody>
</table>
two dice

three dice

\[ f(x \mid \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

for the “unit” normal distribution, 
\( \mu \) (mean) = 0, \( \sigma \) (standard deviation) = 1

\[ f(x \mid 0, 1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \]
Three Dice

• 6 x 6 x 6 = 216 possible throws

• What is the probability of throwing a total of 4?
  • Only three ways: 1-1-2, 1-2-1, 2-1-1
  • $3/216 = 1/72$

• What is the probability of throwing a triplet?
  • $6/216 = 1/36$
Three Dice

- What is the probability of throwing exactly two 6s?

<table>
<thead>
<tr>
<th>1-6-6</th>
<th>2-6-6</th>
<th>3-6-6</th>
<th>4-6-6</th>
<th>5-6-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-1-6</td>
<td>6-2-6</td>
<td>6-3-6</td>
<td>6-4-6</td>
<td>6-5-6</td>
</tr>
<tr>
<td>6-6-1</td>
<td>6-6-2</td>
<td>6-6-3</td>
<td>6-6-4</td>
<td>6-6-5</td>
</tr>
</tbody>
</table>

$\frac{15}{216}$
Three Dice

• What is the probability of not throwing any 6s at all?
  • 5 x 5 x 5 = 125 possibilities, ∴ 125/216

• Probability of exactly one 6?
  • All outcomes must have 0, 1, 2, or 3 6’s
  • Thus, add up all other outcomes and subtract from total
  • 216 - 1 (triplet) - 15 (two) - 125 (none) = 75, ∴ 75/216
The 10 Basic Rules

10) The Chevalier’s Law

• Geeks love to show off

• Ask someone who likes this stuff
Monty Hall

1 2 3
Monty Hall
Games of Pure Luck

Can they be fun?
Put and Take (variant)

• All players start with 10 counters
• All players ante 1 chip per turn
• Roll 1 die:
  • 1 - put 1 counter in middle
  • 2 - put 2 counters in middle
  • 3 - all players put 1 counter in middle
  • 4 - take 1 counter
  • 5 - take 2 counters
  • 6 - take all counters
• Game ends when one player loses all counters
  - player with most counters wins
Boston

- Roll 3 dice - keep highest
- Re-roll other 2 - keep highest
- Re-roll final die, sum total
- Highest score of round wins 1 point for the round
- 2nd round worth 2 points, 3rd worth 3, etc.
- Play 10 rounds (55 points up for grabs)
Spider

• The first player to cross off all numbers (2-12) wins.

• Roll an uncrossed number: cross it off and continue.

• Roll a 7: cross off any number, end turn.

• Roll a crossed number: 1st player to left that can use it crosses it off, end turn.
Betting Games

know your odds...
Playing Environments

- Casino Games - Buy chips, play games run by a “banker” (dealer), sell chips (usually fewer), and leave

- Friendly Games - Meet with friends for fun with small stakes. One player brings chips, players take turn being the banker. See who came out the best

- Family Games - All players have same amount of chips from imaginary bank (chips have no monetary value). Player with most chips at end is the winner
What are the Odds?

- “3 to 1”
- receive banker’s payment in addition to your stake
- “3 for 1”
- how much you’re paid for your bet
- e.g., “5 for 2” = “3 to 2”
True Odds

• The “fair” payout that would keep the game balanced in the long run, without advantage to either side

• Inverse probability of “for” odds

• Chance of rolling a 5 on a die?

• $1/6 = “6 for 1”, or “5 to 1”$

• “For” odds compare favorable outcomes with all outcomes

• “To” odds compare favorable outcomes to unfavorable outcomes
Jolly Seven

- Bet and throw

<table>
<thead>
<tr>
<th></th>
<th>Under 7</th>
<th>7</th>
<th>Over 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 for 1</td>
<td>2 for 1</td>
<td>5 for 1</td>
<td>2 for 1</td>
</tr>
</tbody>
</table>
Analysis

• There are 6 ways to roll a 7, out of 36 possibilities, giving odds of 1 in 6

• True odds are 6 for 1, bank pays 5 for 1, bankers cut is therefore 16.67%

• Half of the remaining 30 rolls are below 7 (likewise for above) = 15/36

• True odds are 12 for 5, bank pays 10 for 5, bankers cut is 16.67%

• Moral: Don’t play. Be the Banker.
<table>
<thead>
<tr>
<th></th>
<th>UNDER 7</th>
<th></th>
<th>OVER 7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 for 1</td>
<td></td>
<td>2 for 1</td>
</tr>
<tr>
<td>7</td>
<td>5 for 1</td>
<td>5 for 1</td>
<td>7 for 1</td>
</tr>
<tr>
<td>6</td>
<td>6 for 1</td>
<td>6 for 1</td>
<td>7 for 1</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>9 for 1</td>
<td>9 for 1</td>
</tr>
<tr>
<td>4</td>
<td>10 for 1</td>
<td>10 for 1</td>
<td>10 for 1</td>
</tr>
<tr>
<td>3</td>
<td>15 for 1</td>
<td>15 for 1</td>
<td>15 for 1</td>
</tr>
<tr>
<td>2</td>
<td>30 for 1</td>
<td>30 for 1</td>
<td>30 for 1</td>
</tr>
</tbody>
</table>

Best Bet / Worst Bet?
Straight Up - 35 to 1
Column Bet - 2 to 1
Dozen - 2 to 1
Red or Black - 1 to 1
Odd or Even - 1 to 1
1 to 18 or 19 to 36 - 1 to 1
Split - 17 to 1
Corner - 8 to 1
Street - 11 to 1
Line - 5 to 1
Top Line - 6 to 1
Liar’s Dice
Liar’s Dice

• Each player rolls 5 concealed dice. Players take turns making claims about all dice at the table.
• In turn, each player has two choices:
  • make a higher bid, or
  • challenge the previous bid (players cannot pass)
• The player may bid an increased quantity of any face, or the same quantity of a higher face.
  • Given a bid of “four fours”, the minimum raise is five of any face, or “four fives”.
• When a bid is challenged, everyone reveals dice. Either the high bidder or the challenger wins the round. The loser loses one die.
• If you have no dice, you are out. Play continues until one player remains.
Homework

• What’s your process for determining your initial bet?

• Why does Liar's Dice start each player with 5 dice? How would other numbers change the dynamics?

• How does the game change as players lose dice? Does this mechanic improve the game? Does it make it more predictable or more unpredictable?